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An update on the performance of reactive energy meters under non-sinusoidal conditions

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Abstract

The different reactive power and energy metering methods used by electric energy meters on medium and high voltage consumers present different results between themselves, notably in the presence of distorted voltage and current signals. Over recent decades, many discussions and various propositions have been put into action, all of which aimed at a worldwide unification of power and reactive energy metering methods. However, the problem still persists and the objective of this study therefore lies in performing of a quantitative update, concerning the impacts of these different methods on the reactive energy metering for billing purposes. In this context, each of the methods frequently used by the electric energy meter manufacturers will be covered in an analytical manner, seeking the mathematical understanding of the divergences that exist between them. In concurrent fashion, calibration tests will be performed in the laboratory aimed at the correct quantification of measurement deviations found under specific distortion conditions in voltage and current waveforms, when comparing meters from different manufacturers and models. A new electric energy meter, that contemplates all known metering methods, was developed with the aim of performing a measurement campaign allowing for the comparison of the performance of each one of these methods under real load conditions, considering many different types of consumer. The obtained results, in addition to providing a quantitative update of the magnitude of existing deviations, also demonstrate a great concern regarding the lack of isonomy currently found around the world in the metering of reactive power and energy.

Keywords Harmonic distortion · Reactive energy and power · Measurement methods · Energy meters

1 Introduction

The billing of electric power and energy for medium and voltage consumers is based not only on active power and energy, but also on reactive power and energy [1]. In this context, it is necessary the use of meter devices that allow for the measuring of these quantities in an isonomic way between the different consumers connected in the electric power systems.

There currently exist various types of power/energy meter on the market, which are based on different metering methods. Under sinusoidal operational conditions, these methods are equivalent, and under the same operational conditions arrive at results that are compatible with the respective classes of accuracy of such device [1].

However, the presence of harmonic components on electric energy systems, discussed in detail in [2, 3], raise questions concerning the accuracy of meters under distorted voltage and current conditions. The main problems are found in the metering of reactive power/energy, as there exist several meters on the market for measuring these electrical quantities, each embedded with different measuring methods [4]. As such when placed under non-sinusoidal conditions, these methods present different values for reactive power and

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energy when subjected to the same voltage and current signals.

In this way, in terms of finances, some consumers may be benefitted, while others are jeopardized in terms of the billing of reactive power and energy.

The scientific community has for decades, brought the question of unification to the fore, in order to define reactive power under non-sinusoidal conditions, and as such different proposals can be found in the literature. Alexander Emanuel, for example presents in [5, 6] formulations for reactive power in the context of harmonic voltage and current components. The definitions presented by Emanuel are the most recent and most commonly accepted by the scientific community, in addition to having been endorsed by IEEE in [7]. Moreover, in [8], the concepts of reactive energy are discussed according to the dictionary of definitions from the IEEE. A number of other power definitions, under distorted voltage and current conditions, were also proposed in other scientific studies [5, 9–18].

Based on the available references, there are clearly two different schools of thought on reactive energy definitions: one considers the 60/50 Hz reactive power as the only significant non-active component of total power and the other aggregates the non-active powers in a single or multiple components of total power. The definition of reactive power, in terms of its physical meaning, becomes a complex task in the light of these two schools of thought. Thus, for the purposes of this work, reactive power will be considered as the part of the total power responsible for maintaining the electric and magnetic fields in the passive energy storage elements of the system, which can be compensated by the simple use of capacitors and reactors. Therefore, the authors in agreement with [5, 6] advocate the first school, which will be used as a reference for comparative purposes throughout the paper.

In addition, it is highlighted herein that the various existing power definitions cannot be confused with the measurement methods used by the different manufacturers of electric energy meters. These methods of measurement are widespread and known to the scientific community. However, the theme lacks any experimental analysis, i.e., the results presented for each method were not amply discussed or analyzed in the context of the impact generated on energy billing, thus showing the relevance, in this way, of the theme discussed in this work.

The objective therefore of the study herein, in terms of scientific contribution, is to identify and present the fragilities and deficiencies of each power and reactive energy metering method, used by the different meters currently available on the market. The study performed also contemplated the development of an electric energy meter that considers, in a simultaneous fashion, the different reactive power and energy measurement methods discussed herein. The referred to meter was used in field measurements in order to quantitatively analyze the discrepancy between the different metering methods studied.

In addition, calibration tests were performed in the laboratory, thus contemplating the different power and energy meters available on the market, aiming at the quantification of the response of different meter devices when submitted to the same distorted voltage and current signals.

2 Theoretical background

Initially, for each of the metering methods considered, an analytical approach was adopted under the intent of understanding the origin of possible metering errors under conditions of distorted voltage and current. To reach this goal, for example, it is necessary to cover the calculation of both active and apparent power, in the case of the power triangle method. In this way, in meters of the electromechanical type, the active power is obtained in continuous form, since the process for obtaining it is based on iterations between the electric and magnetic fields considering for this purpose specific geometric arrangements between the potential and current coils.

Electronic meters, on the other hand, perform discrete sampling of the instantaneous voltage and current values, in a way that one is able to calculate the active power using the values from the discriminated fundamental, dc and harmonic component in the frequency domain, as indicated in (1).

$$P = V_0 I_0 + V_1 I_1 \cos(\theta_1) + \sum_{h=2}^{h\max} V_h I_h \cos(\theta_h) \tag{1}$$

It is important to highlight, according to [7], that significant direct voltage (V_0) and current (I_0) components, in practical terms, are rarely present in alternating current power systems. Therefore, these quantities were not considered in this work.

2.1 Power triangle method

The power triangle method uses rms values for voltage V_{rms} and current I_{rms} to calculate the apparent power S, thus allowing for the calculation of reactive power Q.

Electronic meters record the rms values for voltage and current in discrete form, as indicated in (2) and (3).

$$V_{\text{rms}} = \sqrt{\frac{1}{N}\sum_{n=1}^{N} v_n^2} \tag{2}$$

$$I_{\text{rms}} = \sqrt{\frac{1}{N}\sum_{n=1}^{N} i_n^2} \tag{3}$$

The product between the rms voltage and current, obtained herein, results in the apparent power, as shown in (4).

$$S^2 = V_1^2 I_1^2 + V_1^2\left(\sum_{h=2}^{h\max} I_h^2\right) + \left(\sum_{h=2}^{h\max} V_h^2\right) I_1^2 + \left(\sum_{h=2}^{h\max} V_h^2\right)\left(\sum_{h=2}^{h\max} I_h^2\right) \tag{4}$$

From Eqs. (1) and (4) one determines Q by means of the power triangles. In this way, the behavior of Q, in this method, is directly influenced by P and S. In the following, an analysis is presented that is capable of demonstrating the incapacity of this method to adequately represent reactive power in the presence of harmonic content.

By considering $\varphi_v = \varphi_i$ and $\varphi_{v_h} = \varphi_{i_h}$, that is, voltages and currents in phase for all frequencies, and ignoring dc components, one notes that (1) results in (5).

$$P = V_1 I_1 + \sum_{h=2}^{h\max} V_h I_h \tag{5}$$

However, when calculating reactive power by substituting the values for apparent and active power, indicated in (4) and (5), respectively, one obtains a value for Q different to zero. In this case, the determining of reactive power Q, even though all voltage and current components are in phase, considers the presence of distorted power components, according to the definitions of Alexander Emanuel in [3, 16], in the composition of reactive power. This finding demonstrates an important inconsistency, since in this case the value of Q does not arise from the displacement between voltage and current waveforms on any of the frequencies that constitute the voltage and current signals.

2.2 Current displacement method of 90° by sampling

This method consists of applying a 90° (or $\pi/2$ rad) lag on the current signal to calculate reactive power Q. The mathematical expression of the displaced current is presented in (6).

$$i_d(t) = I_1 \sin\left(\omega t + \varphi_i + \frac{\pi}{2}\right) + \sum_{h=2}^{h\max} I_h \sin\left(h\omega t + \varphi_{i_h} + h\frac{\pi}{2}\right) \tag{6}$$

By performing the calculation for the average value of the power using the product between voltage and displaced current signal, it can be observed that current and voltage components that have different harmonic orders present an average value equal to zero, that is, $Q_{1,h} = Q_{h,1} = Q_{n,m} = Q_{m,n} = 0$. This occurs as the displaced voltage and current are orthogonal.

However, the portions of Q considering currents and voltages with the same harmonic order, contrarily, present an average value different to zero, as indicated in (7).

$$\sum_{h=1}^{h\max} Q_h = \sum_{h=1}^{h\max} V_h I_h \left(\cos\left(\varphi_{v_h} - \varphi_{i_h}\right)\cos\left(h\frac{\pi}{2}\right) + \sin\left(\varphi_{v_h} - \varphi_{i_h}\right)\sin\left(h\frac{\pi}{2}\right)\right) \tag{7}$$

Should a displacement be performed of $-\pi/2$ rad, all the components derived from the product between sines, i.e. the second term of (7), would have signals opposite to those used.

Upon analyzing (7), one notes some peculiarities in terms of the calculation of Q for certain groups of harmonic frequencies. The behavior of reactive power calculated for each one of these groups is described in the following:

Group 1: For $h = 1, 5, 9, 13, \ldots$, or $1 + 4k$, $k = 0, 1, 2, 3, \ldots$, the reactive power can be calculated by (8).

$$Q_h = V_h I_h \sin\left(\varphi_{v_h} - \varphi_{i_h}\right) \tag{8}$$

Group 2: For $h = 2, 6, 10, 14, \ldots$, or $2 + 4k$, $k = 0, 1, 2, 3, \ldots$, the reactive power can be calculated by (9).

$$Q_h = -V_h I_h \cos\left(\varphi_{v_h} - \varphi_{i_h}\right) \tag{9}$$

Group 3: For $h = 3, 7, 11, 15, \ldots$, or $3 + 4k$, $k = 0, 1, 2, 3, \ldots$, the reactive power can be calculated by (10).

$$Q_h = -V_h I_h \sin\left(\varphi_{v_h} - \varphi_{i_h}\right) \tag{10}$$

Group 4: For $h = 4, 8, 12,16, \ldots$, or $4 + 4k$, $k = 0, 1, 2, 3, \ldots$, the reactive power can be calculated by (11).

$$Q_h = V_h I_h \cos\left(\varphi_{v_h} - \varphi_{i_h}\right) \tag{11}$$

In short, Eq. (12) expresses how this method determines the value of reactive power Q.

$$Q = Q_1 - P_2 - Q_3 + P_4 + Q_5 - P_6 + \cdots \tag{12}$$

2.3 Voltage displacement method of 90° by sampling

This method consists of applying a displacement of 90°, or $\pi/2$ rad, to the voltage signal in order to calculate the reactive power Q. The analysis is similar to that of the previous method. Equation (13) shows the mathematical expression of the voltage displacement wave at + 90°.

$$v_d(t) = V_1 \sin\left(\omega t + \varphi_v + \frac{\pi}{2}\right) + \sum_{h=2}^{h\,\max} V_h \sin\left(h\omega t + \varphi_{v_h} + h\frac{\pi}{2}\right) \tag{13}$$

Once again, one notes that only the multiplication between displaced voltage and current of the same frequency present an average value different to zero, thus resulting in (14):

$$\sum_{h=1}^{h\,\max} Q_h = \sum_{h=1}^{h\,\max} V_h I_h \left(\cos\left(\varphi_{v_h} - \varphi_{i_h}\right)\cos\left(h\frac{\pi}{2}\right) - \sin\left(\varphi_{v_h} - \varphi_{i_h}\right)\sin\left(h\frac{\pi}{2}\right)\right) \tag{14}$$

Note here that (14) is similar to (7), where the only distinction is the signal used in components arising from portions constituted of sine functions.

Therefore, analogously to the previous method, the calculated reactive power can contain four different components, depending on the harmonic order. These four different situations are shown in the following.

Group 1: For $h = 1, 5, 9, 13, \ldots$, or $1 + 4k$, $k = 0, 1, 2, 3, \ldots$, the reactive power can be calculated by (15).

$$Q_h = -V_h I_h \sin\left(\varphi_{v_h} - \varphi_{i_h}\right) \tag{15}$$

Group 2: For $h = 2, 6, 10, 14, \ldots$, or $2 + 4k$, $k = 0, 1, 2, 3, \ldots$, the reactive power can be calculated by (16).

$$Q_h = -V_h I_h \cos\left(\varphi_{v_h} - \varphi_{i_h}\right) \tag{16}$$

Group 3: For $h = 3, 7, 11, 15, \ldots$or $3 + 4k$, $k = 0, 1, 2, 3, \ldots$, the reactive power can be calculated by (17).

$$Q_h = V_h I_h \sin\left(\varphi_{v_h} - \varphi_{i_h}\right) \tag{17}$$

Group 4: For $h = 4, 8, 12{,}16, \ldots$, or $4 + 4k$, $k = 0, 1, 2, 3, \ldots$, the reactive power can be calculated by (18).

$$Q_h = V_h I_h \cos\left(\varphi_{v_h} - \varphi_{i_h}\right) \tag{18}$$

The analysis of each case is similar to that presented for the current displacement method. In summary, this method calculates Q according to that indicated by (19).

$$Q = -Q_1 - P_2 + Q_3 + P_4 - Q_5 - P_6 + \cdots \tag{19}$$

Through a close look at Eqs. (12) and (19), one notes that both methods arrive at the same reactive power magnitudes for each harmonic order. However, for odd harmonic orders, the signal is the opposite. In this way, the fundamental reactive power will be inadequately measured. This problem can be avoided by multiplying (19) by − 1, and as such obtaining (20).

$$Q = Q_1 + P_2 - Q_3 - P_4 + Q_5 + P_6 - \cdots \tag{20}$$

2.4 Voltage displacement method by derivative

The voltage displacement method by voltage derivative consists of deriving a voltage signal and calculating the power through the product between the resulting voltage and original current. The derivative of a generic voltage signal in the time domain is given by (21).

$$\frac{\mathrm{d}v(t)}{\mathrm{d}t} = V_1\omega\cos(\omega t + \varphi_v) + V_h h\omega\cos\left(h\omega t + \varphi_{v_h}\right) \tag{21}$$

By using Eq. (21) to calculate Q, one notes that all reactive power components arising from voltage and current of different frequencies contain average values equal to zero. The other components of Q, products from the multiplying of signals of the same frequency, result in (22).

$$Q = V_1 I_1\,\omega\,\sin(\varphi_v - \varphi_i) + \sum_{h=2}^{h\,\max}\left(V_h I_h\,\omega\,h\,\sin\left(\varphi_{v_h} - \varphi_{i_h}\right)\right) \tag{22}$$

By analyzing Eq. (22), one notes the presence of the term ω, or ωh for the harmonic components. As the meters are designed to measure power in the fundamental frequency, the result from (22) is divided by ω. However, for the harmonic components, besides ω, there also exist the order h on the product, which increases the reactive harmonic power value in a way that is proportional to its harmonic order h. In this way, this method calculates Q, according to (23).

$$Q = Q_1 + 2Q_2 + 3Q_3 + 4Q_4 + \cdots \tag{23}$$

2.5 Voltage displacement method by integration

Voltage displacement by integration consists of integrating the voltage signal and calculating the power through the product between the time domain voltage signal integral and the original current signal. The integral of the voltage signal is given by (24).

$$\int v(t)\,\mathrm{d}t = V_1\frac{(-\cos(\omega t + \varphi_v))}{\omega} + V_h\frac{\left(-\cos\left(h\omega t + \varphi_{v_h}\right)\right)}{h\omega} \tag{24}$$

Using (24) to calculate Q, one obtains (25).

$$Q = \frac{V_1 I_1}{\omega} \sin(\varphi_v - \varphi_i) + \sum_{h=2}^{h\max} \left(\frac{V_h I_h}{h\omega} \sin\left(\varphi_{v_h} - \varphi_{i_h}\right) \right) \tag{25}$$

In Eq. (25), one notes that the reactive power value for each order is divided by its respective angular frequencies. In order to correct this error for the fundamental frequency, the electronic meters multiply expression (25) by ω_1. For the fundamental frequency, the error is perfectly corrected. However, for each harmonic frequency, the reactive power remains divided by h, as indicated in (26).

$$Q = Q_1 + \frac{Q_2}{2} + \frac{Q_3}{3} + \frac{Q_4}{4} + \cdots \tag{26}$$

2.6 Fourier series method

This methodology performs the fundamental and the harmonic reactive power calculation for all the harmonic orders present on the voltage and current signals. To this end, the Fourier series is applied to the voltage and current signals in a way that separates the existing frequencies, as indicated by Eq. (27).

$$f(t) = \frac{a_0}{2} + \sum_{h=1}^{h\max} \left(a_h \cos(\omega_h t + \varphi_h) + b_h \sin(\omega_h t + \varphi_h)\right) \tag{27}$$

In this case, $f(t)$ one can represent both $v(t)$ and $i(t)$. Therefore, with the frequencies separated, one can calculate reactive power Q for each frequency, and later algebraically sum together the value of Q for each harmonic order, thus obtaining the value of reactive power that results from the analyzed system.

3 Calibration tests

The objective behind this section is to compare the reactive energy measured by the various meters used by electrical utilities in light of distorted voltage and current signals. Noteworthy here is that the measuring of reactive energy makes up part of consumer billing and directly influences both consumer and utility revenues. In order to perform the calibration tests on meters, the laboratory structure shown in Fig. 1 was employed.

The laboratory structure used is composed of a programmable power source that is capable of synthesizing different voltage and current waveforms, which will be applied to the reactive energy meter under test, as well as a certified precision wattmeter used as a comparison standard.

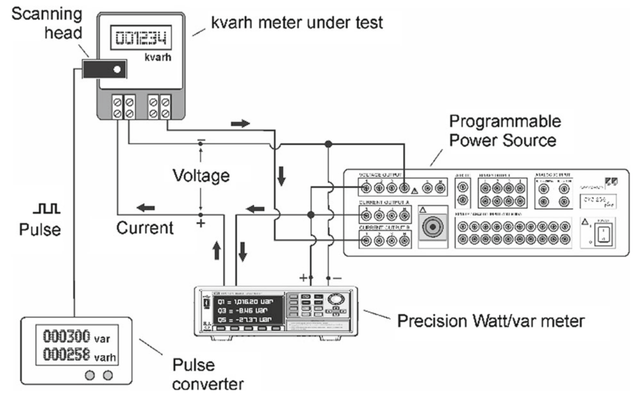

Fig. 1 Calibration test setup diagram

The programmable power source used was the CMC 256 Plus (with an accuracy <0.015%), manufactured by OMICRON Inc., and the precision wattmeter used was the IT9121, manufactured by iTECH. The voltage and current channels of the power source were connected to the reactive energy meter under test, which processes the signals and calculates the reactive energy for billing. Meters of the electronic type emit a luminous pulse that refers to an amount of active energy and another that refers to an amount of reactive energy, each of which is demanded in a given time, denominated as k_h, expressed in *Wh/pulse* and in *varh/pulse*, respectively. In this study, only the reactive energy luminous pulse was used. For this purpose, a specific device was developed for quantifying the measured reactive power, which processes the quantity of luminous pulses during a given period and converts the value of reactive energy into values of reactive power, in accordance with the constant k_h of each meter.

Following this, 19 different meter models for reactive energy and power were used, from 5 different manufacturers. The make and model of each meter were preserved due to industrial secrecy and professional ethics, where the manufacturers were denominated with letters ranging from A to E, and the models were numbered in increasing order for each manufacturer.

Initially, all the meters were submitted to two calibration tests with voltage and current purely sinusoidal. The first test represented a single-phase load with a power factor of 0.70 lagging, while in the second; the power factor of the representative load was 0.70 leading.

In Fig. 2, the results for the tests of each meter are presented. These results are presented as the ratio between the registered reactive power (Q) and the fundamental reactive power (Q_1). Despite the verified deviations, all the results obtained are within the accuracy threshold of meters designed for metering reactive energy and power, equal to 4%. In the following, the meters were submitted to a range of tests involving distorted voltage and current signals. The adopted distortions are commonly found on electric distri-

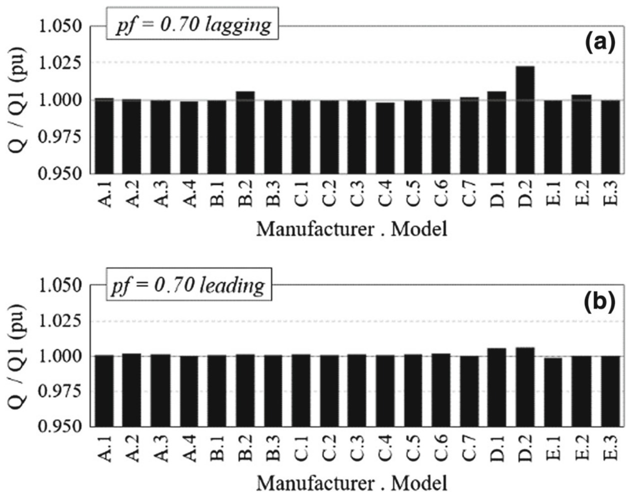

Fig. 2 Relationship between the reactive powers registered by the theoretical reactive power at fundamental frequency

Table 1 Characteristics of the tests performed

#	Characteristics of the tests performed	THD_V (%)	THD_I (%)
1	Odd numbered harmonics not multiples of 3	5.0	10.0
2	Odd numbered harmonics not multiples of 3	7.0	30.0
3	Even harmonics not multiples of 3	7.0	30.0
4	Even harmonics not multiples of 3	9.0	50.0
5	Harmonics multiples of 3	5.0	10.0
6	Harmonics multiples of 3	9.0	50.0

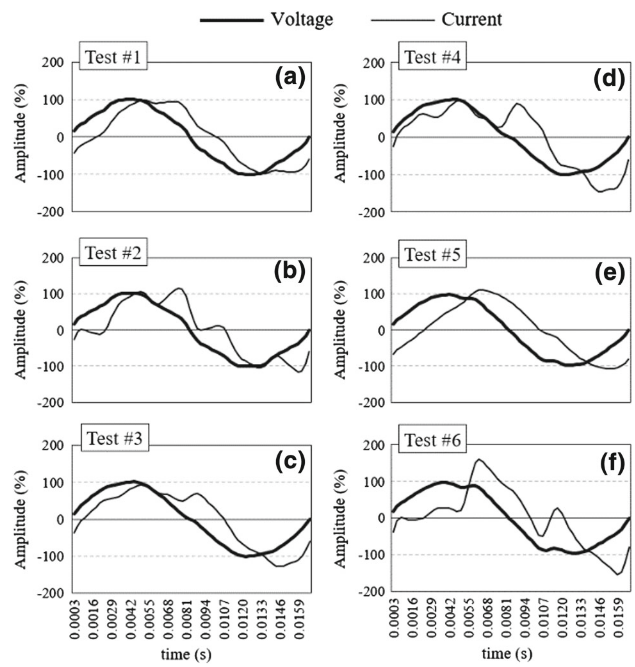

Fig. 3 Voltage and current waveforms for the tests performed

bution networks. In total, 6 tests were performed, with the characteristics presented on Table 1.

By way of illustration, Fig. 3 presents the waveforms of the voltage and current pairs reproduced by the programmable power source for the calibration of meters. Noteworthy here is that on the graphs presented, the vertical axis represents the amplitude as a percentage of the fundamental component for each signal.

The results for reactive power measured by the 19 electronic meters are presented on Fig. 4. Emphasis is here placed upon the fact that it was not possible to determine which method any given meter uses, as this information not divulged by the manufacturer in order to maintain industrial confidentiality. In addition, it is also not possible to analyze which acquisition or processing protocols the individual metering device is using.

Therefore, by analyzing tests #1 and #5, the meters do not present significant divergences between the values recorded and those expected. The largest deviation found was below 1%. Regarding the remaining tests, with exception of 4 models from manufacturer C (C.2, C.3, C.4 and C.5), the deviations found remained below 1.5%.

Through the analysis of meter models C.2 to C.5, which presented the highest deviation discrepancies among the tests performed, one notes deviations from 7.5 to 22.6% for tests #2, #4 and #6. In a general sense, it is noted that the deviations registered present an increase in conjunction with the increase in harmonic distortions, which may be due the different calculation methods for calculating reactive power, as presented in the previous section.

In addition, another experiment performed was that of the calibration of meters under special conditions, where an elementary circuit is considered, constituted of a half wave rectifier, supplying a purely resistive load, as shown in Fig. 5. This figure also shows the voltage waveform (purely sinusoidal) and for the current at the side of the source.

The results obtained by the 19 meters for the reactive power and respective power factor for the half wave rectifier circuit are presented in Fig. 6. Noteworthy here is that the power factor (*fp*) calculated from the fundamental voltage and current components was taken as a reference. Thus, for the specific case of voltage and current signals associated with the half wave rectifier, one finds that the voltage and current in the fundamental frequency are in phase, resulting $fp = 1.0$ pu.

Figure 6 shows that nine meters (47% of the sample) failed to measure the power factor, thus indicating the presence of reactive power (according to the definition presented in Sect. 1), where it should not exist. Based on this result, one can state that the meters considered are based on different methodologies for calculating reactive power. Although it is not possible to precisely identify the methodologies used

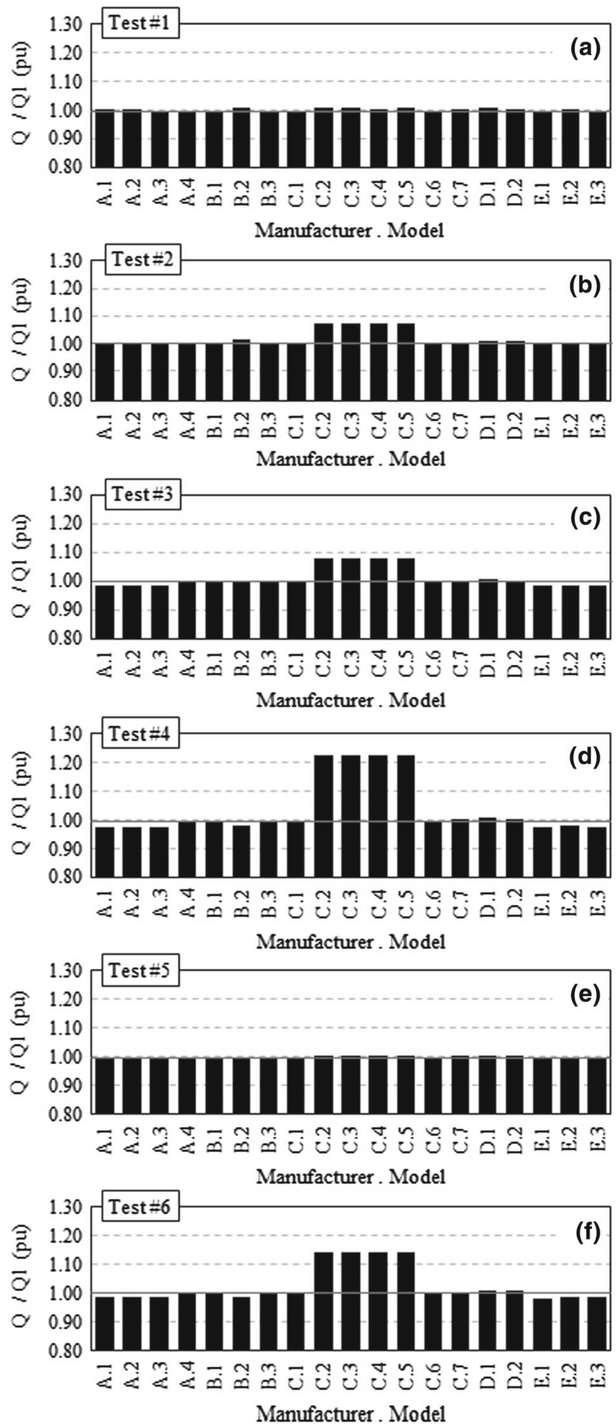

Fig. 4 Relationship between the reactive power registered by the meter and the fundamental reactive power

by each meter, it is observed the existence of meters that consider the harmonic components in the calculation of the reactive power and others that consider only the fundamental voltages and currents.

Therefore, the choice of meter used, and consequently the calculation methodology implemented on it, is a defining factor concerning the billing of reactive energy. Such an affirmation will be validated in the following section.

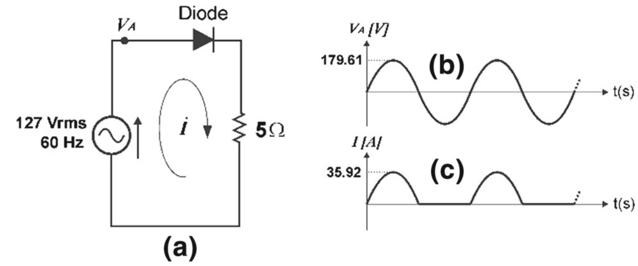

Fig. 5 **a** Test circuit synthesized by the programmable power source, **b** instantaneous voltage and **c** instantaneous current

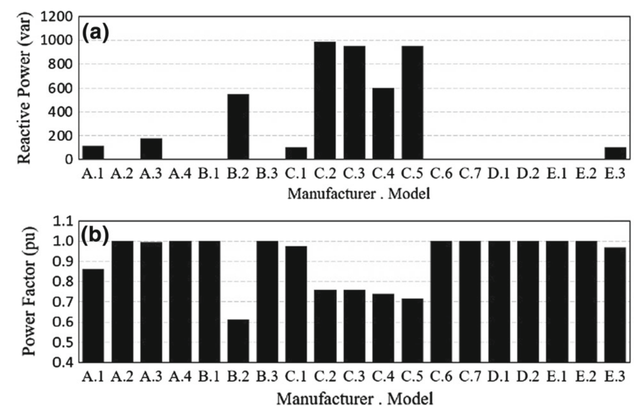

Fig. 6 **a** Reactive power registered by the meter and **b** power factor registered by the meter

4 Metering campaign

Finally, in a way as to quantitatively measure how much the choice of method for calculating reactive power impacts the billing of reactive power and energy, a new three-phase meter was developed that is capable of simultaneously calculating reactive energy using the methods shown in Sect. 2. The developed meter aggregates reactive energy in intervals of 15 min. The developed prototype can be seen in Fig. 7, which presents the following configurations:

- Converter A/D of 16 bits;
- Sampling rate at 128 samples per cycle;
- Indirect metering;
- Operational voltage 120–220 V;
- Current 2.5 (20) A;
- Three-phase four-wire measurement.

The methodologies implemented onto the meter are listed on Table 2 and Fig. 8 shows the schematic diagram of the

Fig. 7 The three-phase meter developed for the study

Table 2 Calculation methodologies implemented for reactive power on the developed meter

Letter	Method
A	Power triangle method (point-to-point)
B	90° Displacement method by sampling (current)
C	90° Displacement method by sampling (voltage)
D	90° Displacement method by derivative (voltage)
E	90° Displacement method by integration (voltage)
F	Fourier method (fundamental only)

algorithms used in the implementation of the listed methodologies.

With the developed meter at hand, 17 m readings were taken for a wide range of consumer profiles of medium and high voltage levels, such as mining facilities, hospitals, television companies, commercial centers, among others. The measurements were performed considering a period of 7 days for each consumer. In addition, it is important to highlight the fact that the reactive power obtained by each of the considered measurement methods was carried out considering the same voltage and current signals in each 15-min aggregation time window. That is, the harmonic spectra of the signal under analysis were exactly the same for the calculation of Q in each of the methods (as shown in Fig. 8), allowing a direct comparison between the results.

The quantitative of reactive energy billed by each of the methods was compared to the reactive energy calculated only from fundamental voltage and current; in accordance with method F. The results are presented on Table 3.

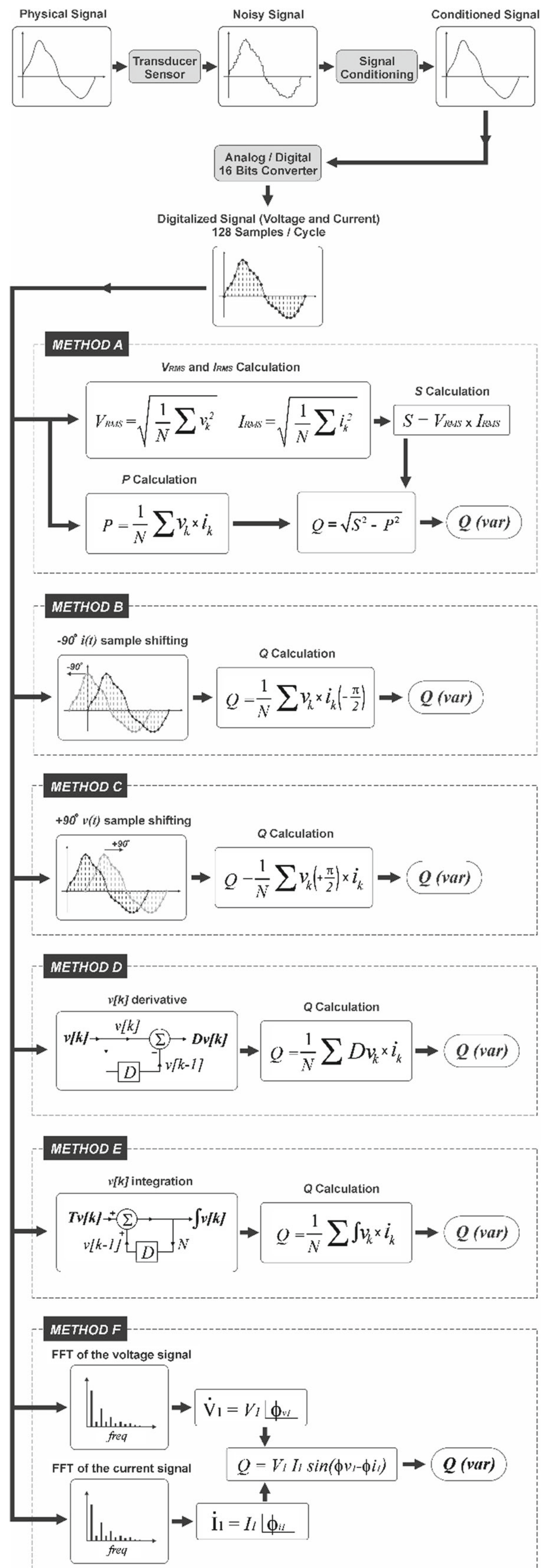

Fig. 8 Schematic diagram of the algorithms used in the meter

Table 3 Reactive energy [in % of Q_{60Hz}] recorded by each metering method considered

Consumer	Method					
	A	B	C	D	E	F*
#1	103.52	99.97	86.30	99.86	100.00	100.00
#2	104.14	99.75	82.39	99.44	99.75	100.00
#3	102.39	99.77	99.76	99.14	99.94	100.00
#4	100.69	99.88	89.88	99.92	99.90	100.00
#5	101.50	99.99	99.97	99.68	100.00	100.00
#6	107.12	99.87	90.26	99.12	99.98	100.00
#7	107.18	99.57	99.55	95.76	99.85	100.00
#8	100.90	99.98	91.54	99.93	100.00	100.00
#9	108.29	99.91	86.91	99.13	99.96	100.00
#10	103.40	100.10	79.39	101.21	100.05	100.00
#11	113.90	100.18	53.55	99.68	100.04	100.00
#12	103.18	100.15	79.10	100.29	100.08	100.00
#13	129.62	94.42	93.82	92.19	95.33	100.00
#14	133.53	99.78	79.54	99.94	100.00	100.00
#15	106.13	99.80	93.92	98.31	99.17	100.00
#16	100.35	99.99	89.79	99.89	100.01	100.00
#17	120.61	95.89	88.43	93.78	94.57	100.00

*The *var* value obtained by using Method F, which results in the value of reactive power measured considering only the fundamental frequency, was used as a basis (100%) for comparing the results obtained by using the other methods

From the analysis of Table 3, one can identify different percentages across all metered consumers. In a general sense, the two methods that present greater discrepancies were method A that always presented reactive energy values higher than those resulting from method F (fundamental frequency) in all cases, and method C, which recorded a lower amount of reactive energy when compared to method F, for all situations.

The highest percentage difference was noted in the metering performed on consumer #11. Also in this meter reading, the highest relative differences were seen in methods A and C, equal to + 13.90% and − 46.45%, respectively; hence, this case was chosen for further study. The meter reading on consumer #11 was performed over 10 days, however, in a way so as to visualize more clearly the differences between the methods, only one day of readings is presented in Fig. 8, which shows the reactive power measured by methods A and F on the same day.

Through the analysis of Fig. 9, one notes that method A presented reactive power higher than method F during the day of the analysis. Noteworthy here is that the first method records as reactive all the components not active with apparent power, i.e., the powers that are the product of the phase shift between voltages and currents of the same order, also those powers arising from the relationship between voltages

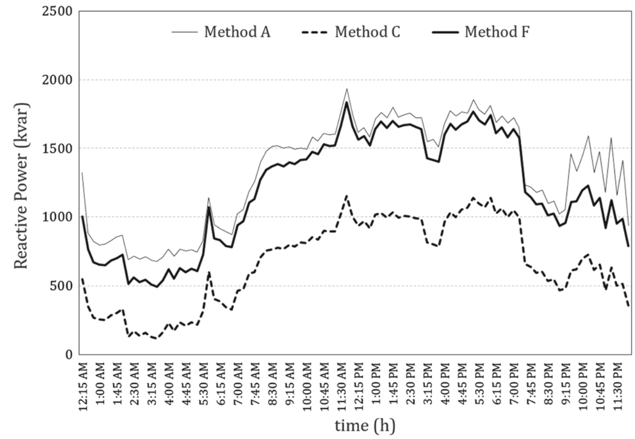

Fig. 9 Reactive power measured by the methods A, C and F

and currents of different frequencies are recorded together, as reactive, as seen in (7).

In contrast, method F calculates as reactive only the power that results from the out of phase product between the voltage and current of the fundamental frequency. Therefore, the conclusion is reached that the power calculated by method A will only be equal to that of method F in systems free of harmonic distortions. On the other hand, the reactive power calculated by method A will always be higher than that calculated by method F.

Different to method A, the reactive power measured by method C was lower than the reactive power measured by method F during the day of the analysis, as shown in the same Fig. 9.

As previously stated, only the frequencies of order $h = 1{+}4k$ possess correct measurements for the energy and reactive power. All the other orders possess phase shifts that are incorrectly altered between the original voltage and current, thus producing an incorrect calculation for the reactive power, as illustrated in (12).

Even with methods A and C presenting results with greater discrepancies in relation to the chosen reference (Method F), that being the fundamental reactive power, all the other methods presented alter the value of reactive power in an incorrect manner, as pointed out in Sect. 2. The methods that displace the voltage and current signals in 90° (B, C, D and E) in particular, are completely inadequate for meter readings in non-sinusoidal environments, since they have a significant impact on the form by which the reactive power of harmonic orders is calculated, each with its own particular methodology. Method A, as it admits the whole portion of non-active power as reactive power, it is considered here as being the product of the displacement of the voltage and current of the same frequency, it is also not recommended for networks with distorted voltages and/or currents. This meter reading methodology will always bring the highest values among those presented. As such, in light of the aforementioned, among the presented methodologies for meter reading, method F is that which records the reactive power in the most correct manner.

Table 3 demonstrates the need for choosing an appropriate calculation methodology and its standardization, since large differences of calculated reactive power, and consequential billing, can occur purely based on the chosen meter.

5 Conclusions

The existence of two different schools of thought on the definitions of reactive power makes the understanding of its physical meaning a very complex task. In any case, based on the reactive power definition considered in Sect. 1, which considers the 60/50 Hz reactive power as the only significant non-active component of total power, the authors chose the results obtained from Method F (fundamental reactive power) as being the most correct from the physical point of view, which was used as a basis for comparing the results obtained by using the other available measuring methods.

In this manner, the studies presented herein, demonstrate that all the methodologies for the measuring of reactive power and energy converge toward the same value, when only signals of fundamental voltage and frequency are considered. However, in the presence of voltage and current distortions, each calculation method behaves in a different way, resulting in discrepancies concerning the amount of calculated reactive power.

In addition to discussing qualitatively the impact that the choice of calculation methodology of reactive power represents in non-sinusoidal conditions, the present study also presented quantitatively the consequences of this choice in commercial meters, used for billing reactive energy. Laboratory tests show large differences of calculated reactive power on the tested meters.

Furthermore, the test with the half wave single-phase rectifier, specifically demonstrated, once again, that there exists a range of calculation methodologies implemented on commercial meters, resulting in different reactive power values for the same voltage and current signal. The consequence of these different means of how meters quantify reactive power is the non-isonomic billing of reactive power and energy on the part of the electrical utility.

Finally, in complement to the laboratory results, a measuring campaign was performed considering a new meter, specially developed for this study, capable of calculating reactive power using various methodologies of calculations simultaneously. Once again, evidence was given that demonstrated that the Q calculation methodology is relevant for its quantification in environments with harmonic distortions. The differences between the measured reactive energy on some consumers can reach more than 50%.

In light of the aforementioned, the need for standardization became evident for the methods of measuring reactive power and energy, in a way that the billing processes of reactive power and energy are performed in an isonomic way for the most diverse types of consumers.

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